

Quantum point contact conductance in NINS junctions

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The effect of an insulating barrier located at a distance a from a NS quantum point contact is analyzed in this work. The Bogoliubov de Gennes equations are solved for NINS junctions (S: anisotropic superconductor, I: insulator and N: normal metal), where the NIN region is a quantum wire. For $a \neq 0$, bound states and resonances in the differential conductance are predicted. These resonances depend on the symmetry of the pair potential, the strength of the insulating barrier and a . Our results show that in a NINS quantum point contact the number of resonances vary with the symmetry of the order parameter. This is to be contrasted with the results for the NINS junction, in which only the position of the resonances changes with the symmetry.

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I. INTRODUCTION

In high critical temperature superconductivity the symmetry of the pair potential is one the most studied aspects [1, 2]. Tunneling spectra depend strongly on this symmetry and therefore tunneling spectroscopy is a very sensitive probe for its study. In a d -symmetry and (110) orientation, for instance, the differential conductance has a peak at zero voltage, called zero-bias conductance peak (ZBCP) which has been predicted theoretically by different works [3, 4, 5, 6, 7, 8] and observed experimentally [9, 10, 11, 12, 13, 14]. The existence of the ZBCP is due to the formation of Andreev bound states at the Fermi level (zero energy states) near to the interface [15, 16, 17]. These states appear due to the interference between scattering quasiparticles at the interface and the sign change of the pair potential. Studies of quantum point contacts in NIS junctions show that the ZBCP is removed by the quasiparticle diffractions at the point contact [18, 19], an aspect that has been shown experimentally [14]. Recently two quantum point contacts have been studied for the crossed Andreev reflection in d -wave superconductors [20].

On the other hand, in NINS [21, 22] and NISN junctions [23, 24], resonances in the differential conductance appear. In anisotropic superconductors, the resonance energies depend as well on the symmetry of the pair potential. In NINS junctions and d_{xy} -symmetry, e.g., the positions of these resonances are out of phase with respect to those predicted for isotropic superconductors [25] and in NISN junctions the conductance presents two kinds of resonances due to anisotropy of the pair potential [24].

In this paper, we analyze the differential conductance

when quasiparticles are injected into a superconductor from a single-mode quantum wire, with an insulating barrier located at a distance a of the NS interface (NINS quantum point contact). We show that there exist bound states which cause resonances in the differential conductance and that the number of these resonances depends on the symmetry of the order parameter. This is shown through the solution of the Bogoliubov-de Gennes equation in NINS junctions, where NIN region is modeled by a wire of width W . In particular s and d - symmetries are considered.

II. THE BOGOLIUBOV-DE GENNES EQUATION AND ITS SOLUTIONS IN NINS POINT CONTACTS

The elementary excitations or quasiparticles in a superconductor are described by the Bogoliubov de Gennes (BdG) equations, which can be generalized for anisotropic superconductors [26]. For a steady state these equations are

$$\begin{aligned} H_e(\mathbf{r}_1)u(\mathbf{r}_1) + \int d\mathbf{r}_2 \Delta(\mathbf{r}_1, \mathbf{r}_2)v(\mathbf{r}_2) &= Eu(\mathbf{r}_1), \\ -H_e^*(\mathbf{r}_1)v(\mathbf{r}_1) + \int d\mathbf{r}_2 \Delta(\mathbf{r}_1, \mathbf{r}_2)u(\mathbf{r}_2) &= Ev(\mathbf{r}_1), \end{aligned} \quad (1)$$

where $H_e = -\hbar^2 \nabla^2 / 2m + V(\mathbf{r}) - \mu$ is an electronic hamiltonian and μ the chemical potential. $\Delta(\mathbf{r}_1, \mathbf{r}_2)$ is the pair potential, $u(\mathbf{r}_1)$ and $v(\mathbf{r}_1)$ are the wave function for the electron- and hole-like components of a quasiparticle,

$$\psi(\mathbf{r}) = \begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix}. \quad (2)$$

It is supposed that the quasiparticle moves on the x - y plane, the interfaces are normal to the x -axis and the NIN region has a width W in the y direction, see fig.

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1. The insulating barrier is modeled by a delta function, $V(x) = U_0\delta(x+a)$. The solutions of the BdG equations in the N_I , N_{II} and in the superconducting regions, are respectively,

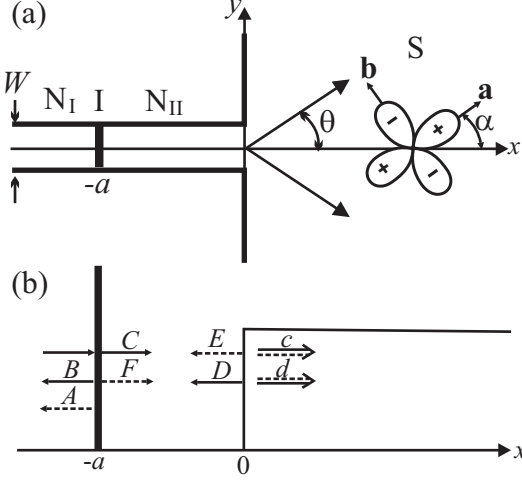


FIG. 1: (a) The point contact NINS junction, the insulating barrier is located in $x = -a$ and the NIN region is a single mode quantum wire with width W . (b) Scattering processes, the solid line and dashed line represent the electron and the hole-like components of a quasiparticle respectively.

$$\psi_{N_I} = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_1^+ x} + A \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_1^- x} + B \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_1^+ x} \right] \phi_1(y) \quad (3)$$

$$\psi_{N_{II}} = \left[C \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_1^+ x} + D \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_1^+ x} + E \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_1^- x} + F \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-ik_1^- x} \right] \phi_1(y), \quad (4)$$

$$\psi_S = \int_{-k_F}^{k_F} ds \left[c(s) \begin{pmatrix} u_0^+(s) e^{i\varphi_+(s)/2} \\ v_0^+(s) e^{-i\varphi_+(s)/2} \end{pmatrix} e^{ik_+^+(s)x} + d(s) \begin{pmatrix} v_0^-(s) e^{i\varphi_-(s)/2} \\ u_0^-(s) e^{-i\varphi_-(s)/2} \end{pmatrix} e^{-ik_-^-(s)x} \right] e^{isy}, \quad (5)$$

where

$$k_1^\pm = \sqrt{k_F^2 \pm 2mE/\hbar^2}, \quad k_1 = \sqrt{k_F^2 - \pi^2/W^2}, \quad k_\pm^\pm(s) = \sqrt{k_1^2 \pm 2m\Omega_\pm(s)/\hbar^2}. \quad (6)$$

$$\phi_1(y) = \sqrt{\frac{2}{W}} \sin \left[\frac{\pi}{W} \left(y + \frac{W}{2} \right) \right], \quad (7)$$

$$\Omega_\pm(s) = \sqrt{E^2 - |\Delta_\pm(s)|^2}, \quad (8)$$

$$u_0^\pm(s) = \sqrt{\frac{1}{2} \left[1 + \frac{\Omega_\pm(s)}{E} \right]}, \quad v_0^\pm(s) = \sqrt{\frac{1}{2} \left[1 - \frac{\Omega_\pm(s)}{E} \right]}. \quad (9)$$

The quasiparticles with k_+^+ and k_-^- wavenumber move in the pair potential Δ_+ and Δ_- respectively and given by

$$\Delta_\pm(s) = \Delta(\pm k_\pm^\pm \hat{i} + s \hat{j}) \equiv \Delta_\pm e^{i\varphi_\pm},$$

with

$$\Delta(\mathbf{k}) = \int d(\mathbf{r}_1 - \mathbf{r}_2) e^{i\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2)} \Delta(\mathbf{r}_1 - \mathbf{r}_2). \quad (10)$$

All the evanescent modes have been neglected. This approximation is justified due to the fact that for $\pi < Wk_F < 2\pi$ the narrow wire is a single mode and the energy of evanescent modes is well above the Fermi energy [18, 20]. One finds A, B, C, D, E, F, c and d using boundary conditions in $x = -a$ and $x = 0$. The electron-electron and electron-hole reflection coefficients are respectively

$$R_e = \left| \frac{h}{g} \right|^2, \quad R_h = \left| \frac{2F_3}{g} \right|^2, \quad (11)$$

where

$$g = (1 + Z^2) [(1 + F_1)^2 - F_2 F_3] + Z^2 [(1 - F_1)^2 - F_2 F_3] e^{-2i(k_+ - k_-)a} + Z [1 - F_1^2 + F_2 F_3] [Z(e^{2ik_+a} + e^{-2ik_+a}) + i(e^{-2ik_+a} - e^{2ik_+a})], \quad (12)$$

$$h = (F_1^2 - F_2 F_3 - 1) [Z^2 e^{2i(k_+ + k_-)a} - (1 - iZ)^2] - Z(Z + i) [2F_1(e^{2ik_+a} - e^{2ik_-a}) + (1 + F_1^2 - F_2 F_3)(e^{2ik_+a} + e^{2ik_-a})], \quad (13)$$

$$F_i = \frac{4}{\pi^2 \sqrt{\gamma_F^2 - 1}} \int_{-\gamma_F}^{\gamma_F} dq \frac{\sqrt{\gamma_F^2 - q^2}}{(1 - q^2)^2} \cos^2[\pi q/2] f_i(q), \quad (14)$$

$$f_1 = \frac{1 + \Gamma_+ \Gamma_- e^{-i(\varphi_+ - \varphi_-)}}{1 - \Gamma_+ \Gamma_- e^{-i(\varphi_+ - \varphi_-)}}, \quad f_2 = \frac{2\Gamma_- e^{i\varphi_-}}{1 - \Gamma_+ \Gamma_- e^{-i(\varphi_+ - \varphi_-)}}, \quad f_3 = \frac{2\Gamma_+ e^{-i\varphi_+}}{1 - \Gamma_+ \Gamma_- e^{-i(\varphi_+ - \varphi_-)}}, \quad (15)$$

$$\Gamma_\pm = \frac{v_0^\pm}{u_0^\pm}, \quad \gamma_F = \frac{k_F W}{\pi} \quad \text{and} \quad Z = \frac{m U_0 \gamma_F}{\hbar^2 m \sqrt{\gamma_F^2 - 1}}. \quad (16)$$

III. DIFFERENTIAL CONDUCTANCE

Using the BTK model [27], the normalized differential conductance, G_R , is calculated from

$$G_R = \frac{G_S}{G_N} = \frac{[(1 + F_0)^2 + 4Z^2](1 - R_e + R_h)}{4F_0}, \quad (17)$$

where G_N the conductance when $\Delta = 0$ and $a = 0$. F_0 is defined by (14) with $f_i = 1$. For d -symmetry $\Delta_{\pm} = \Delta_0 \cos(2(\theta \mp \alpha))$, α is the angle between the (100) axis of the superconductor and the normal to the interface, and $\theta = \sin^{-1}(s/k_F)$. (cf. fig. 1a.)

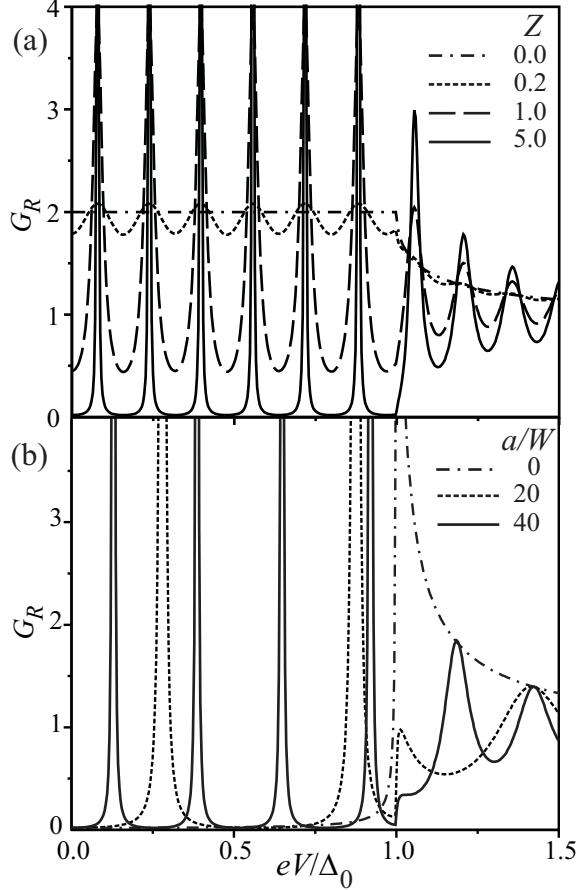


FIG. 2: Differential conductance for s -symmetry. (a) Different values of Z with $a = 63W$; (b) different values of a with $Z = 5$. In both cases $k_F W = 1.7$.

Figures 2 and 3 show the differential conductance for s and $d_{x^2-y^2}$ -symmetries. When $a = 0$ (NIS point contact), our results agree with [18]. For $a \neq 0$, subgap resonances appear in the differential conductance and their number increases with a . When Z decreases, it can be seen that the number and position of the resonances remain unchanged, only the peaks become broader and for $d_{x^2-y^2}$ -symmetry the broad is greater for a fixed Z value.

Figure 4 exhibits the differential conductance for $\alpha = \pi/4$ (d_{x-y} -symmetry). ZBCP does not appear because

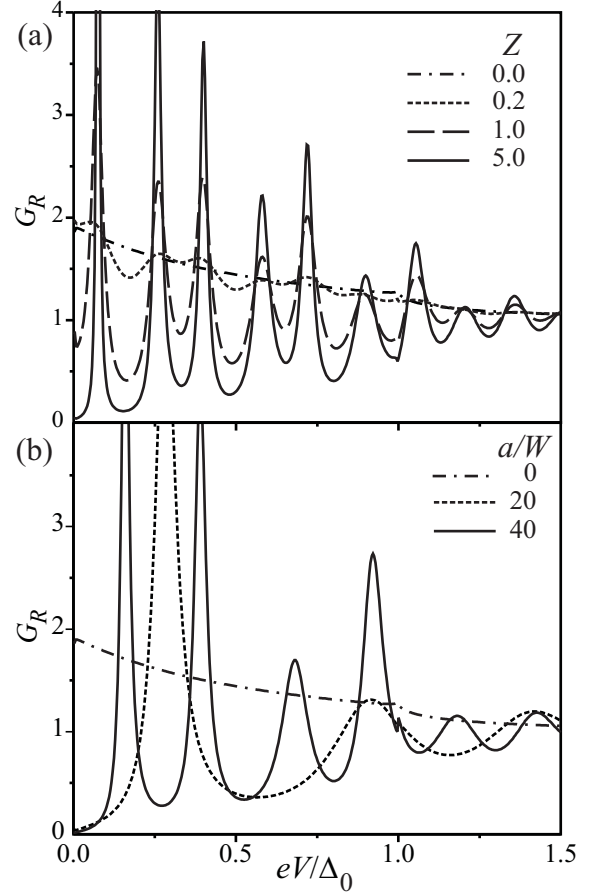


FIG. 3: Same as in fig. 2 for $\alpha = 0$ ($d_{x^2-y^2}$ -symmetry).

its Andreev reflection is zero. In the last case the wave functions in the channel are a superposition of two plane waves with wave numbers $k_y = \pm\pi/W$, each wave experiences a pair potential phase 0 and π respectively and therefore the Andreev reflection coefficient $a(\theta)$ ($R_h(\theta) = |a(\theta)|^2$) for each wave are opposite, the waves of the reflected holes interfere destructively and the Andreev reflections vanish. In relation with the d_{x-y} -symmetry the number of resonances decreases compared with the s and $d_{x^2-y^2}$ -symmetries. Additionally when Z decreases the number of resonances is constant and the peak broadens and its position is smoothly displaced toward the right.

The subgap resonances in the differential conductance are a direct consequence of the quasibound states formed inside the energy gap. The energies and lifetime of these bound states are given by the poles of the current transmission amplitude. Setting $g = 0$ in Eq.12 one finds these poles. A complex energy, $E = E_R + iE_I$, is required in order to solve this equation, where E_R is the position of resonance and $\hbar/2|E_I|$ is the lifetime of the quasibound states.

The resonance positions E_R for s or $d_{x^2-y^2}$ -symmetries

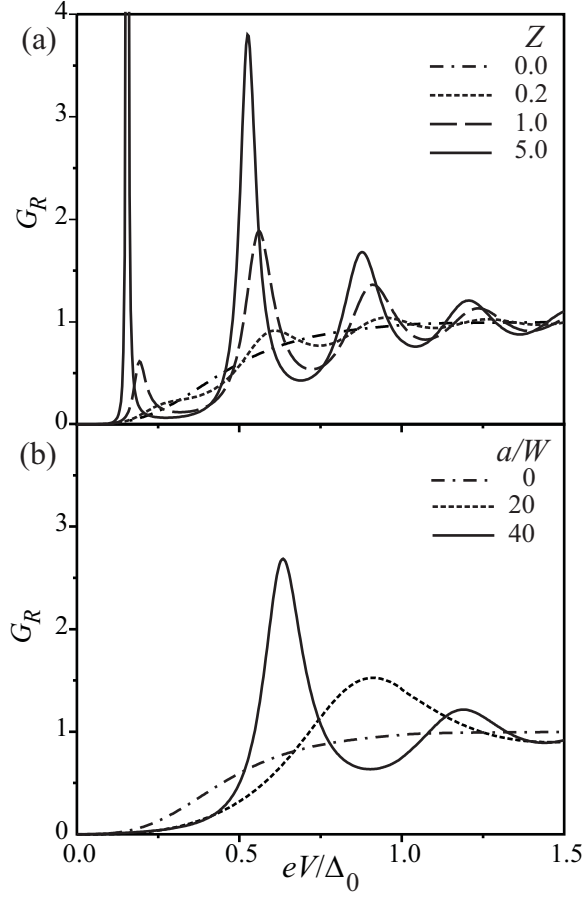


FIG. 4: Same as in fig. 2 for $\alpha = \pi/4$ (d_{x-y} -symmetry).

are given by

$$E_n = E_0(n\pi - \phi), n = 1, 2, \dots, \quad (18)$$

and for d_{xy} symmetry are determined from

$$E_n = E_0(2n\pi - \phi'), n = 1, 2, \dots \quad (19)$$

In these equations

$$E_0 = \frac{E_F}{a} \sqrt{1 - \gamma_F^{-2}} \quad (20)$$

and ϕ, ϕ' are phases that depend on Z, a and E . Therefore the number of resonances with $E < \max(\Delta)$ for s or $d_{x^2-y^2}$ -symmetries are approximately twice the corresponding number of a d_{xy} -symmetry. This is due to the fact that in the case of a d_{xy} -symmetry the Andreev reflection is zero. Thereby the quantization of the bound states occurs when the quasiparticles travel in a closed path a distance equal to $2a$ in the x direction and $E_n \propto 2n\pi/2a$. (The quasiparticle is transmitted as an electron in $x = -a$ and reflected as an electron in $x = 0$.) In the case of s - or $d_{x^2-y^2}$ -symmetries the quasiparticles complete a closed path when they travel a distance $4a$

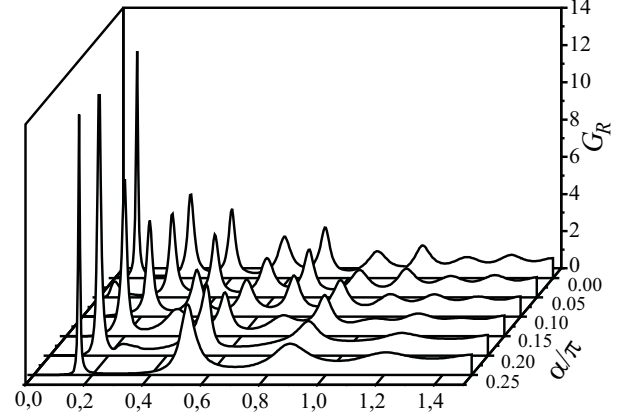


FIG. 5: Differential conductance for different values of α with $Z = 5, a = 63W$ and $k_F W = 1.7$.

in x direction and $E_n \propto 2n\pi/4a$. (The quasiparticle is transmitted in $x = 0$ as an electron, reflected as a hole in $x = 0$, reflected as a hole in $x = -a$ and finally reflected as an electron in $x = 0$.) Therefore one has that in this case the number of the quasibound states is approximately twice the corresponding number of d_{x-y} symmetry.

In order to determine the lifetime of the quasibound states, a semiclassical argumentation will be used. The lifetime τ is defined as the time that a quasiparticle in the INS region requires to "scape" toward the N_I or S regions. For the s or $d_{x^2-y^2}$ -symmetries the time that a quasiparticle needs for around trip is

$$T = \frac{4a}{\hbar k_{0F1}/m} = \frac{2\hbar d}{E_F \sqrt{1 - \gamma_F^{-2}}}. \quad (21)$$

If N is the number of closed trips, τ is given by

$$\tau = TN, \quad (22)$$

N is obtained from

$$[R_{e-h}R_{h-e}R_{h-h}R_{e-e}]^N = 1/e, \quad (23)$$

where R_{e-h}, R_{h-e} are the electron-hole and hole-electron reflection coefficients respectively for $Z = 0$ (point contact NS) and RI_{e-e}, RI_{h-h} the electron-electron and hole-hole reflection coefficients respectively for an insulating barrier (IN). From equations (21), (22) and (23) the lifetime is obtained as

$$\tau = -\frac{\hbar d}{E_F \sqrt{1 - \gamma_F^{-2}}} \frac{1}{\ln(Z^2 R_{e-h}/(1 + Z^2))}, \quad (24)$$

where we have used the fact that $R_{e-h} = R_{h-e}$ and $RI_{e-e} = RI_{h-h} = Z^2/(1 + Z^2)$. Equation (24) is similar to that found for NINS junction with s -symmetry

[21]. Similarly for the d_{xy} -symmetry the lifetime is given by

$$\tau = -\frac{\hbar d}{E_F \sqrt{1 - \gamma_F^{-2}} \ln(Z^2 R_{e-e}/(1 + Z^2))}, \quad (25)$$

with R_{e-e} the electron-electron reflection coefficient for $Z = 0$. For the case of s -symmetry, and $E < |\Delta|$, $R_{e-h} = 1$. Therefore the lifetime increases with Z and tends to infinity for $Z \gg 1$, while the resonance width, $2|E_I| \approx \hbar/\tau \rightarrow 0$, as is observed in fig. 2. For $d_{x^2-y^2}$ -symmetry the quasiparticles transmission is finite for $E < \Delta_0$ due to the anisotropy of the pair potential, $R_{e-h} < 1$, and the lifetime increases with Z but is finite for $Z \gg 1$. This is observed in the width of the resonances in fig. 3. For the d_{xy} -symmetry the behavior of the lifetime and the width of the resonances are similar to the case of $d_{x^2-y^2}$ -symmetry, see Fig. 4. For all cases, with $E > \Delta_0$, the reflection coefficients are always less than one, the lifetimes decrease and the widths of the resonances increase.

Figure 5 shows G_R for different values of α , when α change from 0 to $\pi/4$, some peaks begin to decrease and vanish for $\alpha \approx 0.20\pi$. This happens because the Andreev reflections decrease and the electron-electron reflection increases. For $\alpha = \pi/4$ the Andreev reflections are zero and one has the conductance for d_{xy} -symmetry. Similarly the values of the energy of the resonances move toward the left as α increases due to a change of the phase ϕ in the solution of the equation $g = 0$.

IV. CONCLUSIONS

Our results show that in NINS point contacts the differential conductance have resonances due to bound states. The number of resonances depends on the symmetry of the order parameter, in contrast to a NINS junction. In the latter case only the position of the resonances changes with the symmetry. The number of resonances with $E < \max(\Delta)$ (subgap resonances) for s or $d_{x^2-y^2}$ -symmetries is approximately twice the corresponding number of the d_{xy} -symmetry. When α change from 0 to $\pi/2$ some peaks disappear because to Andreev reflection vanishes.

In the case of s -symmetry, the lifetime of quasibound states increases with the insulating barrier strength and is infinite for $Z \gg 1$. In contrast, for a d -symmetry the lifetime increases with Z but is finite for $Z \gg 1$. This occurs because the quasiparticles transmission is different of zero for $E < \Delta_0$ in contrast to the case of s -symmetry, where the transmission is zero for $E < \Delta_0$. Therefore the lifetime of the resonances decreases in d -symmetries and the width of the resonances increases. The results obtained in this work can be used to find the symmetry of high temperature superconductors in experiments of the type carried out in references [10] and [11].

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- [1] C. C. Tsuei and J. R. Kirtley, Phys. Rev. Lett. **85**, 182 (2000).
 - [2] D. J. V. Harlingen, Rev. Mod. Phys. **67**, 515 (1995).
 - [3] Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. **74**, 3451 (1995).
 - [4] S. Kashiwaya and Y. Tanaka, Rep. Prog. Phys. **63**, 1641 (2000).
 - [5] S. Kashiwaya, Y. Tanaka, M. Koyanagi, H. Takashima, and K. Kajimura, Phys. Rev. B **51**, 1350 (1995).
 - [6] S. Kashiwaya, Y. Tanaka, M. Koyanagi, and K. Kajimura, Phys. Rev. B **53**, 2667 (1996).
 - [7] Y. S. Barash, A. A. Svidzinsky, and H. Burkhardt, Phys. Rev. B **55**, 15282 (1997).
 - [8] M. B. Walker and P. Pairor, Phys. Rev. B **60**, 10395 (1999).
 - [9] M. Covington, R. Scheuerer, K. Bloom, and L. H. Greene, Appl. Phys. Lett. **68**, 1717 (1996).
 - [10] L. Alff, H. Takashima, S. Kashiwaya, N. Terada, H. Ihara, Y. Yanaka, M. Koyanagi, and Kajimura, Phys. Rev. B **55**, R14757 (1997).
 - [11] J. Y. T. Wei, N.-C. Yeh, D. F. Garrigus, and M. Strasik, Phys. Rev. Lett. **81**, 2542 (1998).
 - [12] E. P. B. N. M. Aprili, M. Covington and L. H. Greene, Phys. Rev. B **57**, R8139 (1998).
 - [13] W. Wang, M. Yamazaki, K. Lee, and I. Iguchi, Phys. Rev. B **60**, 4272 (1999).
 - [14] I. Iguchi, W. Wang, M. Yamazaki, Y. Tanaka, and S. Kashiwaya, Phys. Rev. B **62**, R6131 (2000).
 - [15] J. Yang and C.-R. Hu, Phys. Rev. B **50**, 16766 (1994).
 - [16] C.-R. Hu, Phys. Rev. Lett. **72**, 1526 (1994).
 - [17] Y. Tanaka and S. Kashiwaya, Phys. Rev. B **53**, 9371 (1996).
 - [18] Y. Takagaki and K. H. Ploog, Phys. Rev. B **60**, 9750 (1999).
 - [19] K. Tsuchikawa, N. Yoshida, Y. Tanaka, S. Kashiwaya, J. Inoue, and Y. Takagaki, Physica C **352**, 224 (2001).
 - [20] S. Takahashi, T. Yamashita, and S. Maekawa, cond-mat/0401111.
 - [21] R. A. Riedel and P. F. Bagwell, Phys. Rev. B **48**, 15198 (1993).
 - [22] S. H. Tessmer and D. J. V. Harlingen, Phys. Rev. Lett. **70**, 3135 (1993).
 - [23] W. L. McMillan, Phys. Rev. **175**, 559 (1968).
 - [24] W. J. Herrera and J. V. Ni3o, phys. stat. sol.(b) **220**, 555 (2000).
 - [25] J. H. Xu, J. H. Miller, Jr., and C. S. Ting, Phys. Rev. B **53**, 3604 (1996).
 - [26] C. Bruder, Phys. Rev. B **41**, 4017 (1990).
 - [27] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).